## Objectives:

The purpose of this experiment is to investigate two different methods of performing vector addition: graphically (using the tip-to-tail method) and analytically (using the method of components).

## Theory:

A vector is a quantity that possesses both a magnitude and a direction. A vector may be represented by drawing a straight along the action of the quantity, the length of the line being made proportional to the magnitude of the vector, with an arrowhead placed at the end of the line to show the direction. Some examples of vector quantities include displacement, velocity, acceleration, and force. Mathematical operations can be performed with vector quantities (i.e. addition and subtraction), and the result of the operations are called the resultants. For purposes of this lab, we will be adding two or more vectors together and the sum will be our resultant.

Let's consider the particular vector quantity of force. If two or more forces act at a point, a single force may act as the equivalent of the combination of forces. The resultant in this case is a single force which produces the same effect as the sum of several forces. In Figure 1 below, vectors $\vec{A}$ and $\vec{B}$ add together to produce the resultant vector $\vec{R}$. A vector may also be broken up into components. The components of a vector are two vectors in different directions, usually at right angles, which will give the original vector when added together (see Figure 2). In this case, vector $\vec{R}$ has been broken into a horizontal component ( $\vec{R}_{x}$ ) and a vertical component ( $\vec{R}_{y}$ ).


Figure 1: Two vectors, their resultant, and their equilibrant


Figure 2: A vector and its components

The operation of adding vectors graphically consists of constructing a figure in which a line is drawn from some point of origin to represent the first vector, then from the arrowhead end of this line and at the proper angle with respect to the first vector, another line is drawn to represent the second vector, and so on with the remaining ones (this is also called the tip-to-tail method). The resultant is the vector drawn from the origin of the first vector to the arrowhead of the last (see Figure 3 where R is the resultant vector). If a closed polygon is formed by the vectors being added, then the resultant is zero; and if these vectors represent forces, the object being acted upon is in equilibrium.


Figure 3: Graphical vector addition using the tip-to-tail method
Vectors may also be added analytically by calculating the $x$ and $y$ components of each vector, getting the algebraic sum of all the $x$ components and the algebraic sum of all the $y$ components where $R_{x}=\left(A_{x}+B_{x}\right)$ and $R_{y}$ $=\left(A_{y}+B_{y}\right)$ (Figure 4 may help you understand why this works).


Figure 4: Analytical vector addition using the method of components

## Procedure:

1. Vector $\vec{A}$ has a magnitude of 5 cm with a direction of $20^{\circ}$. Another vector $\vec{B}$ has a magnitude of 10 cm with a direction of $120^{\circ}$. Using the graph paper provided, draw a vector diagram to scale and determine graphically the magnitude and direction of the resultant vector $\vec{R}=\vec{A}+\vec{B}$. Set the origin of your coordinate system at the center of the graph paper.
```
Resultant magnitude =
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Resultant direction $=$
2. On a new sheet of graph paper, redraw vectors $\vec{A}$ and $\vec{B}$ and draw a new vector $\vec{C}$ that has a magnitude of 7.5 cm with a direction of $220^{\circ}$. Determine graphically the magnitude and direction of the resultant vector $\vec{R}=\vec{A}+\vec{B}+\vec{C}$. Set the origin of your coordinate system the center of the graph paper.

```
Resultant magnitude =
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Resultant direction $=$
3. On a new sheet of graph paper, draw a vector with a magnitude of 10 cm and a direction of $30^{\circ}$. Find the magnitude of the components of this vector by graphically determining the length of its $x$ component and its $y$ component. Set the origin of your coordinate system at the center of the graph paper.

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x-component magnitude =
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y -component magnitude $=$

## Calculations:

1. Calculate the resultant in Procedure 1 analytically using the method of components. Compare this result with your graphical result. Use a percent error calculation to quantify the difference between the graphical and analytical vector magnitudes and directions.
2. Calculate the resultant in Procedure 2 analytically using the method of components. Compare this result with your graphical result. Use a percent error calculation to quantify the difference between the graphical and analytical vector magnitudes and directions.
3. Calculate the $x$ and $y$ components of the vector in Procedure 3 analytically using the method of components. Compare this result with your graphical result. Use a percent error calculation to quantify the difference between the graphical and analytical vector magnitudes and directions.
