

## THE BALLISTIC PENDULUM

### THEORY:

Newton's second law of motion states that the time rate of change of momentum is equal to the net force. In a collision between two objects each one exerts a force on the other. These forces are equal and opposite, hence the net force acting on the system at impact is zero and the total momentum of the system remains unchanged. Momentum is therefore conserved in a collision, regardless of the nature of the collision.

During the collision the objects become deformed and a certain amount of energy is used to deform them. If the objects are not elastic, they will remain permanently deformed and energy used to deform them is not recovered. Inelastic impact can be illustrated by the ballistic pendulum in which a ball is fired into a pendulum bob and remains embedded in it. If the pendulum bob is initially at rest and the projectile strikes the pendulum with a velocity  $\underline{v}$ , then conservation of momentum requires that  $m\underline{v} = (m+M)\underline{V}$ , where  $m$  is the mass of ball,  $M$  is the mass of pendulum bob and  $\underline{V}$  is the velocity of bob with embedded ball after collision.

As a result of the collision the pendulum with embedded ball swings about its point of support and the bob rises to a height  $h$  above its initial position. By the law of conservation of energy, we therefore have  $\frac{1}{2}(M+m)V^2 = (M+m)gh$  where  $V$  is the velocity magnitude (speed) of bob with embedded ball as before. This equation gives a way of determining  $V$  since it results in  $V^2 = 2gh$ . By substituting this value of  $V$  and the values of the masses  $m$  and  $M$  into the momentum equation, the value  $v$  of the speed of the ball may be determined. This method is one which is often used to determine the velocity of a rifle bullet with a heavy block of wood as a pendulum bob.

The velocity of the ball can also be determined by observing and measuring projectile motion. Measurements of the range and vertical distance of fall when the bullet is fired horizontally give velocity in terms of range  $x$ , distance of fall  $y$ , and the acceleration of gravity. Since the horizontal component of velocity is constant, then  $x = vt$ , where  $x$  is the distance,  $v$  is the speed and  $t$  is the time of fall. The initial vertical component of velocity is zero, and hence the motion in that direction is the same as a freely falling body with zero initial velocity.  $y = \frac{1}{2}gt^2$ , where  $g$  is the acceleration of gravity.

The apparatus used in this experiment is a combination of ballistic pendulum and spring gun. The pendulum bob consists of a massive cylinder hollowed out to receive the ball. This bob is suspended by a strong lightweight rod pivoted at its upper end. The projectile is a brass ball which is fired into the pendulum bob and held there by a spring. An index is attached to the pendulum bob to indicate the location of the center of mass of the loaded pendulum. When the projectile is fired into the bob, the pendulum swings upward and is caught at its highest point by a pawl which engages a tooth in the curved rack.

### EQUIPMENT:

|                    |              |
|--------------------|--------------|
| Ballistic pendulum | meter sticks |
| Protractor         | plump bob    |

### PROCEDURE:

1. The velocity of the projectile is measured first by means of measurements of range and fall. Therefore the pendulum is swung up onto the rack and hooked with the pawl so that it will not interfere with the free flight of the ball. (It may be necessary instead to remove the pendulum and the rack from the apparatus. If so, see procedure 7.) The apparatus should be set up near the corner of the table in a reproducible location. The gun is cocked by pushing against the ball and compressing the spring until the trigger is engaged. The ball is to be fired horizontally and therefore the apparatus must be level. As a preliminary run, the ball is fired and the location of impact on the floor determined approximately. Several shots may be necessary to make this estimate. Be sure to check placement of apparatus before each shot. Have a catcher located in a place where he can catch the ball after one bounce on the floor.

2. Tape a sheet of white paper to the floor at the approximate point of impact of the ball. When the ball strikes the paper, it will leave a mark on it. In this way an exact record can be obtained of the spot where the ball strikes the floor. This sheet of paper is your data, and the distance of its edge closest to the gun should be measured and marked. Use the plumb bob to locate the point on the floor directly below the ball as it leaves the gun.
3. Fire the ball five times and calculate the range for each shot. The range is the horizontal position from the point of projection to the point of impact. Record these distances in your data table.
4. Repeat 1-3, this time using the 2<sup>nd</sup> gun engagement setting. (The gun has 3 settings).
5. Measure the vertical distance of fall, i.e., the vertical distance of the point of projection above the floor.
6. Determine the center of gravity of the pendulum with embedded ball by removing the pendulum from the stand (be careful not to lose the nut) and balancing the pendulum with ball on a sharp edge. (This point may already be marked from previous uses, but you should confirm it.)
7. While the pendulum is removed from the stand, measure and record its mass (without the ball). Also measure and record the mass of the ball.
8. Release the pendulum from the rack and allow it to hang freely. With the pendulum at rest, fire the ball into the pendulum bob and determine the number of the notch on the curved scale reached by the pawl when it catches the pendulum. To remove the ball from the pendulum, push the spring catch up and push ball out with finger. Do this four more times.
9. From these measurements compute the average value of the position of the pendulum on the rack. Set the pendulum with the pawl engaged in the notch which corresponds most closely with this value. Use a protractor to measure the angle  $\theta$  with respect to the vertical; be sure that the protractor measures "0" in the vertical position. With the pendulum hanging in its lowest position, measure the distance  $L$  from the center of the pivot point to the center of gravity.
10. Repeat Procedures 8-9 with the second setting.

**CALCULATIONS: Do these for each gun setting.**

1. Compute the average range of the projectile.
2. From the measured vertical distance of fall and the known value of  $g$ , calculate the time of flight of the ball.
3. Compute the velocity of projection for each setting from the results obtained in calculations 1-2.
4. From the data of procedures 5-8 compute the vertical distance  $h (= L - L \cos \theta)$  through which the center of gravity of the loaded pendulum was raised as a result of the collisions.
5. Using  $h$ , compute the value of the velocity of the pendulum ball and bob just after collision.
6. By use of the law of conservation of momentum, calculate the velocity of the ball before collision.

7. Using the data of your experiment, calculate
  - a. the kinetic energy of the ball just before impact
  - b. the kinetic energy of the loaded pendulum bob just after impact
  - c. the loss of energy during this impact.
  
8. Calculate the fractional loss of energy during this impact.

QUESTIONS:

1. What became of the energy that was lost (you calculated the fraction in Calculation 8.)?
  
2. Compute the ratio of the mass of the pendulum bob to the loaded pendulum bob. How does this ratio compare to the fraction of energy lost during the impact (Calculation 8)?
  
3. A bullet of mass 10.0 g is fired horizontally into a block of wood of mass 2000 g and suspended like a ballistic pendulum. The bullet sticks in the block and causes the block to swing so that its center of gravity rises 10.0 cm. Find the velocity of the bullet just before impact. This method is often used to determine the muzzle velocity of a rifle bullet.