PENDULUMS & PERIODIC MOTION

THEORY:

A simple pendulum consists of a point mass suspended from a stationary point by a massless string in a uniform gravitational field. Such a pendulum will execute periodic motion which is very nearly simple harmonic motion if the angular amplitude is small. At larger angles it is still periodic, but differs from simple harmonic. While the uniform gravitational field is experimentally realistic, the point mass and massless string are a matter of scale: instead of a point mass, the length of the mass should be small compared to the overall length of the pendulum and the mass of the string should be small compared to the mass attached to the string.

The amplitude of the vibration should not affect the period of simple harmonic motion, but for pendulum motion, the period is affected. The simple harmonic period for a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where T is the period in seconds; l is the length of the spring from the suspension point to the point mass in meters; and g is the magnitude of the gravitational acceleration in meters per second squared.

APPARATUS:

String
Support for suspending the pendulum
Hooked mass
Timer
Meter stick

PROCEDURE:

1. Measure and cut a piece of string about a meter long. Tie a loop in one end for the hooked mass.

2. Set the support arm about 1 meter above the table top. Attach the mass to the string, and tie the string to the support arm so that the pendulum is between 70 and 80 cm long.

3. Measure the distance from the support point to the topmost cylindrical "edge" of the mass.

4. Time 10 small angle oscillations (less than 10 degrees). Do this 5 times, recording each time separately. Be sure that you count the oscillations correctly (When you start the watch, don't say "one"!)

5. Time 10 oscillations with a starting angle of 50 degrees (roughly). Do this 5 times, recording each time separately.

6. Shorten the pendulum to between 50 and 60 cm and collect data as in steps 3-5.

7. Shorten the pendulum to between 30 and 40 cm and collect data as in steps 3-5.

CALCULATIONS:

1. From the data of Procedures 2-5 and 6 and 7 calculate the periods for each measurement. Then calculate the means and standard-deviations for the small-angle periods and the large-angle periods for each length.

2. From the small-angle mean periods determine the effective length of each pendulum by assuming the oscillations are simple harmonic.

3. For each length calculate the % difference of lengths: (measured - effective)/ measured X 100%.

4. Calculate the ratio of the large-angle period to small-angle period for each length.

5. Comment on the ratios based on the following table:

Table 1. Ratio of theoretical period to simple harmonic period for simple pendulum

Angle amplitude (degrees)	0	10	20	30	40	45	50	60
Ratio	1.000	1.002	1.008	1.017	1.0313	1.0400	1.0498	1.073

6. Calculate the following test statistic for each length. It will help show whether the difference in the small-angle and large-angle oscillations is statistically significant.

$$t = \frac{\overline{T_{large}} - \overline{T_{small}}}{\sqrt{\dfrac{s_{large}^2 + s_{small}^2}{5}}}$$

If this value is larger than 2.306, the data support a real difference at the 95% confidence level. What does your data say?

7. Plot a graph using the values of the measured lengths as abscissas and the small-angle periods as ordinates. The best curve through these points should NOT be a straight line. Use the Graphical Analysis program to fit a **power curve** to this data. Be sure to turn off "connecting lines" and use "Autoscale to 0" on both axes. You may need to "assist" the program by suggesting that the exponent is close to 0.5.