Algebra of functions

consider: $f(x) = x^2 - 4$ $g(x) = x+2$
and $h(x) = 3x - 7$

Find:

$f(-2)$

$$f(-2) = (-2)^2 - 4$$
$$= 4 - 4$$
$$= 0$$
$$f(-2) = 0$$

Find:

$f+g$

there are different ways to combine functions. $f+g$ ask us to write a new function that represents the function f plus the function g.

that is

$$(f+g)(x) = f(x) + g(x)$$

Steps to work: $f+g$

1st: know that $f+g$ means $f(x)+g(x)$

$$(f+g)(x)=f(x)+g(x)$$

2nd: write what each function is.. that is $f(x)$ equals what, $g(x)$ equals what.

$$(f+g)(x) \quad \overbrace{x^2-4}^{f(x)} + \overbrace{x+2}^{g(x)}$$

3rd: Simplify as much as possible.

$$(f+g)(x)= x^2-4+x+2 = x^2+x-2$$

$$= x^2+x-2$$

usually write in decending order i.e. highest exponent first

Find:

$g-h$

on all of these problems follow the same basic steps. The difference is the operation.

$g-h$ means combine the functions g and h together by subtraction.

1st so $(g-h)$ $g(x)-h(x)$

notice the ( ). These were place here to remind us that not just $3x$ is being subtracted but $3x-7$ is being subtracted.

2nd $(g-h)(x) = x+2-(3x-7)$

$(g-h)(x) = x+2-3x+7$

$= -2x+9$

See page 104

for a function f and a function g

$(f+g)(x) = f(x)+g(x)$ $\quad$ $(f-g)(x) = f(x)-g(x)$

$(fg)(x) = f(x)g(x)$ $\quad$ $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

Find: fh

1st $(fh)(x) = f(x)h(x)$

2nd $(fh)(x) = (x^2-4)(3x-7)$

3rd $\left\{\begin{array}{l} = (x^2-4)(3x-7) \\ = 3x^3-7x^2-12x-28 \end{array}\right.$

Find: gh

1st $(gh)(x) = g(x)h(x)$

2nd $= (x+2)(3x-7)$

3rd $\left\{\begin{array}{l} = 3x^2-7x+6x-14 \\ = 3x^2-x-14 \end{array}\right.$

Find $\frac{f}{g}$

1st $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

from College A
$x^2-4 = (x+2)(x-2)$
F.O.I.L to confirm

2nd $= \frac{x^2-4}{x+2}$

3rd $= \frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}} = (x-2)$

The composition of two functions.

When a composite of two functions is made the range of one function becomes the domain of the second function. Said another way the out puts of one function become the inputs of the other function.

This is how that looks algebraically.

$$(g \circ f)(x) = g(f(x))$$

means composite

see page 10C

ex. Let $f(x) = x-3 \quad g(x) = x^2+5$

$$(g \circ f)(x) = g(f(x))$$

remember $g(x) = x^2+5$

so

$$g(f(x)) = (f(x))^2 + 5$$

$$= (x-3)^2 + 5$$

$$= x^2 - 6x + 9 + 5$$

$$= x^2 - 6x + 14$$

Let $h(x) = 2x$ and $f(x) = 3x^2 - 2x - 7$

find $f \circ h$

$$(f \circ h)(x) = f(h(x))$$

remember

$$f(x) = 3x^2 - 2x - 7$$

so $f(h(x)) = 3(h(x))^2 - 2(h(x)) - 7$

$$= 3(2x)^2 - 2(2x) - 7$$

$$= 3 \cdot 4x^2 - 4x - 7$$

$$= 12x^2 - 4x - 7$$